The Similarities and Differences between the Aristotelian Relations and the Duality Relations: From the Traditional Square of Oppositions to the Buridanian Octagons[1]

Mingyu MA

(Catholic University of Leuven)

Author: Mingyu MA, PhD candidate of Catholic University of Leuven, Tel: +8613984053530, Email: mamingyuleon@163.com

Abstract: This article attempts to deal with two sets of logical relations, namely Aristotelian relations and duality relations. Through classical diagrammatic representations assigned to them, we get the Aristotelian square of oppositions and the duality square which seem to have a sort of isomorphism. As a matter of fact, many interesting Aristotelian squares turn out to also be duality squares, and vice versa. The aim of this article, however, is to show that the two sets of relations in question are neither equivalent nor isomorphic. They not only have a variety of different logical properties, but are also mutually independent both in essence and conceptually. By adding more formulas, the diagrams exhibiting the logical relations between these formulas became more complicated and the differences between the Aristotelian diagrams and the duality diagrams have become more perspicuous. One of these complicated diagrams that has been chosen for this article is the octagon provided by Buridan, the great 14th century logician. This octagon is both an extension of the opposition square and the duality square. As a result, the non-isomorphism between Aristotelian relations and duality relations embodied in this one and single representation is more easily and clearly perceived.

Key Words: Aristotelian relations; Duality relations; Traditional square of oppositions; Duality square; Buridanian octagon

1. Introduction

This article attempts to deal with two sets of logical relations which areknown in the literatures by the names of the Aristotelian relations and the duality relations. The purpose of this discussion is to gain a deeper and better understanding of each by inspecting one with reference to another. The name of the former set of relations has an apparent historical connotation as the four types of logical relations classically incorporated under its head appear for the first time in Aristotle's logical works. In the later development of that theory, however, the explications and applications of those relations by some of the main logicians and philosophers go far beyond the original linguistic and conceptual

[1] Cf., Paulos Huang, "Dialogue and Critique: The 16th Century Religious Reform and Modernity", International Journal of Sino-Western Studies, vol. 12, 1-12. (https://www.sinowesternstudies.com/back-issuses/vol-12-2017/)

framework offered by Aristotle. Hence whenever the term Aristotelian is used, it is not intended to mean that the following concept or proposition modified by it is historically proposed or used by Aristotle himself. Rather it is merely a mark or a tag indicating a certain well-defined logical relation in question.

Duality is a very general and pervasive phenomenon encountered in nearly every branch of mathematical theories and other theories that have mathematically formalized languages to some extent. Generally speaking, the principle of duality associates one object, concept, structure or theorem with another one. It is a principle about how two concepts or operations can be substituted with each other in a variety of contexts. Probablythe most popular and simple example of the general notion of duality comes from set theory. The various kinds of operations involved in complementation typically represent the duality phenomenon. Suppose that both the sets A and B are subsets of a given set E. That is to say, all the complementation operations (') are to be performed relative to E. Then we will have the following facts:

① $\varnothing' = E$ and $E' = \varnothing$

② $A \cap A' = \varnothing$ and $A \cup A' = E$

③ $A \supseteq B$ iff $B' \supseteq A'$

④ $(A \cup B)' = A' \cap B'$ and $(A \cap B)' = A' \cup B'$

It is quite easy to see that all the equations appear in pairs. This highlights the syntactic feature of the notion of duality, which mainly concerns the possibility of making substitutions of certain logical operations in our formulas. The formulas resulting from the substitutions become the dual formulas of the original ones.[2] In particular there is one kind of duality phenomenon pervasive in various natural and artificial languages which concerns the interaction between external negation and internal negation. It is this duality that is most frequently discussed in linguistic, logical and philosophical contexts and specifically it is this duality that people have in mind as they are thinking about the Aristotelian square of oppositions. Furthermore, there is an interesting phenomenon in the history of logic that various concrete squares proposed and considered by logicians of different era are both Aristotelian squares and duality squares embodying the interactions between the negation operations.

The most apparent and prominent non-logical feature shared by Aristotelian relations and duality relations is that they are both given certain diagrammatic representations. Doubtlessly the most popular one is the so called traditional square of oppositions and its corresponding duality square. In middle ages, several important logicians all pay their attentions to the square of

[2] Paul Halmos, Naive Set Theory, (Springer, New York, 1974), 17-18.

oppositions, such as Avicenna (Chatti 2012, 2014)[3] and Buridan (Hughes 1987; Read 2012).[4] At the same time, in order to investigate more complex logical systems and syllogism patterns, the medieval logicians start to go beyond the restrictive area of squares and 4 propositions. They take so many more propositions and logical relations into account that the resulting diagrams become extraordinarily complicated, such as the hexagon of William of Sherwood (Kretzmann 1966; Khomskii 2012)[5] and 3 complex octagons of Buridan. In contemporary formal logic, the square of oppositions is widely used in a variety of branches, like general modal logic (Fitting and Mendelsohn 1998; Carnielli and Pizzi 2008)[6], epistemic logic (Lenzen 2012)[7] and temporal logic (Rini and Cresswell 2012)[8]. At the same time, the opposition square is also applied far outside of the boundary set by logic and philosophy into the field of natural language, psychology, neural science, computer science and so on. Just as the logician Jacquette said, the opposition square and the logical relations demonstrated in it have already become a certain type of lingua franca used in various different domains.[9]

Similar situation obtains in the case of duality. The ubiquity of the phenomenon in most of the formal languages following the laws of classical logic and the logical behaviors of it are already well-known. In natural language, duality exists cross-linguistically and systematically and the realizations of which are much more diverse and complicated. These data about duality have led to the view that it would be better to be treated as a type of semantic universal.[10] Hence duality and duality square also indeed play the role of lingua franca no less than the Aristotelian relations. In virtue of focusing on duality, we are hopefully able to achieve a better understanding of the interactions between natural languages and logics on one side and their formal counterparts on the other.

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[3] S. Chatti, "Logical Oppositions in Arabic logic: Avicenna and Averroes", in Béziau, J. Y. & Jacquette, D. eds., Around and Beyond the Square of Opposition, (Springer, Basel, 2012), 21-42. And Chatti, "Avicenna on possibility and necessity", Hist. Philos. Log. 35, 2014, 332-353.

[4] G. Hughes, "The modal logic of John Buridan," in Corsi, G., Mangione, C. & Mugnai, M. eds., Atti del convegno internazionale di storia della logica le teorie delle modalita, (CLUEB, Bologna, 1987), 93-111. S. Read, "John Buridan's theory of consequence and his octagons of opposition." In Béziau J. Y. & Jacquette, D. eds., Around and Beyond the Square of Opposition, (Springer, Basel, 2012), 93-110. For the historical origin and development of the traditional square of opposition, see: T. Parsons, "The traditional square of opposition," in Zalta, E. N. ed., Stanford Encyclopedia of Philosophy, (CSLI, Stanford, 2017); P. Seuren, The Logic of Language, (Oxford University Press, Oxford, 2010), Chapter 5; D. Londey and C. Johanson, The Logic of Apuleius, (Brill, Leiden, 1987); M. Correia, "Boethius on the square of opposition," in Béziau, J. Y. & Jacquette, D. eds., Around and Beyond the Square of Opposition, (Springer, Basel, 2012), 41-52.

[5] N. Kretzmann, William of Sherwood's Introduction to Logic, (University of Minnesota Press, Minneapolis, 1966). Y. Khomskii, "William of Sherwood, singular propositions and the hexagon of opposition", in Béziau J. Y. & Jacquette, D. eds., Around and Beyond the Square of Opposition, (Springer, Basel, 2012), 43-60. For the modern version of this hexagon, see: T. Czeżowski, "On certain peculiarities of singular propositions," Mind, 64, (1955), 392-395.

[6] M. Fitting & R. Mendelsohn, First-Order Modal Logic, (Kluwer, Dordrecht, 1998). W. Carnielli & C. Pizzi, Modalities and Multimodalities, (Berlin, Springer, 2008).

[7] W. Lenzen, "How to square knowledge and belief," in Béziau J. Y & Jacquette, D. eds., Around and Beyond the Square of Opposition, (Springer, Basel, 2012), 305-311.

[8] A. Rini & M. Cresswell, The World-Time Parallel. Tense and Modality in Logic and Metaphyiscs, (Cambridge University Press, Cambridge, 2012).

[9] D. Jacquette, "Thinking Outside the Square of Opposition Box," in Béziau J. Y & Jacquette, D. eds., Around and Beyond the Square of Opposition, (Basel, Springer, 2012), 81.

[10] van Benthem, "Linguistic universals in logical semantics", In: Zaefferer, D. ed., Semantic Universals and Universal Semantics, (Foris, Berlin, 1991), 17-36.

The first apparent impression we get by inspecting the Aristotelian square and the duality square side by side is that they seem to have an isomorphism to some extent. As a matter of fact, a good deal of interesting Aristotelian squares turns out to be duality squares simultaneously and vice versa. In spite of that, the two sets of relations in question are actually neither equivalent nor isomorphic. They have not only a variety of different logical properties but are also mutually independent both essentially and conceptually. It is not appropriate to characterize one set of logical behaviors in terms of languages and concepts suitable for the other. At the level of a square, as simple as it is, the inappropriateness is not that prominent. With the addition of more formulas, however, the geometric figures exhibiting the logical relations among them are getting more complicated and consequentially the differences between the Aristotelian diagrams and the duality diagrams are getting more perspicuous. By reference to those mutual discrepancies, it is easier to shed light on the nature of each one of them separately. The complicated diagram chosen in this article is the octagon provided by Buridan, the great 14th century logician. This octagon is both an extension of the opposition square and the duality square. As a result, the non-isomorphism between the Aristotelian relations and the duality relations embodied in this one and single representation is more clearly perceived.

This article then follows the following structure: in section 2, duality is dealt with. After the presentation of some typical cases of duality relations, a formal definition is given, the logical properties are characterized and the duality squares are drawn; in section 3, the Aristotelian relations are expounded both formally and informally. The corresponding oppositions squares are drawn at last; in section 4, the structural similarities and the essential differences of the Aristotelian relations and the duality relations are discussed in two parts respectively. In section 5, the focus is on Buridan's modal octagon, the purpose of which is to make the differences between the two sets of logical relations much more prominent.

2 Duality and its definition

2.1 Some typical cases of duality

As has been said in introduction, the duality phenomenon concerned with in this article comes from the interactions between two kinds of negation. In this section, some simple and concrete examples are presented.

In propositional logic, we have the following four equivalences about the logical behaviors of conjunction and disjunction:

(1) $\varphi \wedge \psi \equiv \backsim (\backsim \varphi \vee \backsim \psi)$

(2) $\varphi \vee \psi \equiv \backsim (\backsim \varphi \wedge \backsim \psi)$

(3) $\backsim (\varphi \wedge \psi) \equiv \backsim \varphi \vee \backsim \psi$

(4) $\backsim (\varphi \vee \psi) \equiv \backsim \varphi \wedge \backsim \psi$

The four equivalents make it clear that the external negation of conjunction (disjunction) equals the internal negation of disjunction (conjunction) and the external negation of the internal negation of conjunction (disjunction) equals disjunction (conjunction). Taken together, the above four equivalents themselves are equivalent to each other and say the same thing, that is, the conjunction and disjunction connectives are each other's dual.

Another classical example comes from predicate logic, which concerns the logical interaction between the universal quantifier and the existential quantifiers. Again, we have four equivalents about the logical relations between universal and existential quantifiers:

(5) $\exists x\varphi \equiv \sim \forall x \sim \varphi$

(6) $\forall x\varphi \equiv \sim \exists x \sim \varphi$

(7) $\sim \exists x\varphi \equiv \quad x \sim \varphi$

(8) $\sim \quad x\varphi \equiv \exists x \sim \varphi$

Analogouslythe four equivalents indicate that the universal quantifier stands in the duality relation to the existential quantifier.

Finally in modal logic and between the necessity operator and possibility operator, the similar equivalents obtain once again:

(9) $\Box\varphi \equiv \sim \quad \sim\varphi$

(10) $\quad \varphi \equiv \sim \Box \sim\varphi$

(11) $\sim \Box\varphi \equiv \quad \sim\varphi$

(12) $\sim \quad \varphi \equiv \Box \sim\varphi$

2.2 General definitions and logical properties of duality

Duality can be definedon different levels of abstraction. Based on the typical examples above, the logical operators involved will be generalized to any n-ary operator. Assuming A to be the set of all well-formed formulas, $\sim$ to be the negation connective in classical logic and O, Q to be two n-ary operators on set $A: A^n \rightarrow A$, then for any n number of formulas $a_1, \cdots, a_n$ in A, the duality relations can be defined as follows:

- O is the external negation of Q $(E(O,Q))$ iff $O(a_1, \cdots, a_n) \equiv \sim Q(a_1, \cdots, a_n)$
- O is the internal negation of Q $(I(O,Q))$ iff $O(a_1, \cdots, a_n) \equiv Q(\sim a_1, \cdots, \sim a_n)$
- O is the dual of Q $(D(O,Q))$ iff $O(a_1, \cdots, a_n) \equiv \sim Q(\sim a_1, \cdots, \sim a_n)$

The duality examples from2.1c clearly show that any group of duality phenomenon actually involves not only two propositions or operators, but four. Given a proposition P, negating it externally, internally and both, we will get a group of four propositions including the original one. Some logicians, therefore, recommend replacing the notion of duality with the notion of quaternality (Gottschalk 1953).[11] In order to technically tackle the four propositions in a uniform manner, it would be better to view the proposition P itself as a result of some operation, which is called identity (ID) and defined as follows:

- $O = Q$ (ID(O,Q)) iff $O(a_1, \cdots, a_n) \equiv Q(a_1, \cdots, a_n)$

As aresult, we get a set of duality relations containing four elements: $D = \{ID, E, I, D\}$. It is easy to check that the elements in $\mathcal{D}$are not just any old relations but functions. For any operator O and for any relation R in D, there exists a unique operator Q such that $R(O,Q)$. As long as the functionality of the duality relations is guaranteed, we are able to change our notation from relation to function and relative to the set D, for any operator Q we are able to form another four-element set

[11] W. H. Gottschalk, "The theory of quaternality," Symb. Log. 18, (1953), 193-196.

of duality operators generated by Q: $O(Q)=\{ID(Q), E(Q), I(Q), D(Q)\}$. By investigating the above concrete duality examples, simple logical computations reveal the following properties of the set D:

a) $E \circ E(Q) = \mathrm{ID}(Q) = Q$

b) $I \circ I(Q) = \mathrm{ID}(Q) = Q$

c) $D \circ D(Q) = \mathrm{ID}(Q) = Q$

d) $E \circ I(Q) = I \circ E(Q) = D(Q)$

e) $E \circ D(Q) = D \circ E(Q) = I(Q)$

f) $D \circ I(Q) = I \circ D(Q) = E(Q)$

Informally speaking, for any operator, the external negation of its external negation, the internal negation of its internal negation and the dual of its dual are all equivalent to itself. And among E, I and D, any combination of any two of them equal the third. The first three clauses together confirm that all the duality relations have the property of symmetry. From the above equations, it is not difficult to see that the set $O(Q)$ actually can be generated by any one of its element. Generally speaking, for any operator $Q' \in O(Q)$, we have $O(Q') = O(Q)$, which implies that $\mathcal{O}(Q)$ is closed under the duality relations. Applying any operation in $\mathcal{D}$ in any number of times to any operator in $O(Q)$ will not take us outside of the set.[12] In their concrete linguistic realizations, the relevant logical behaviors are not expressed merely by one single proposition or sentence but a set of logically equivalent propositions.

As so many scholars have already pointed out (Piaget 1949; Gottschalk 1953; Loebner 1990; van Benthem 1991; Peters and Westerståhl 2006; Demey and Smessaert 2015), the property of duality can be clearly shown by a Klein four group V4 with the following Cayley table:

$\circ$	ID	E	I	D
ID	ID	E	I	D
E	E	ID	D	I
I	I	D	ID	E
D	D	I	E	ID

2.3 Duality squares

According to the above definitions of duality relations, based on the set of duality operators $O(Q)$ generated by any Q and its logical properties, we can visually represent the duality relations as follows (INEG stands for internal negation; ENEG stands for external negation; DUAL stands for dual.):[13]

[12] Westerståhl formulates this property as a fact about the duality square, that is, each quantifier in the square spans the same square. See: D. Westerståhl, "Classical vs. modern squares of opposition, and beyond," in Béziau, J. Y. & Payette, G. eds., The Square of Opposition: A General Framework for Cognition, (Peter Lang, Bern, 2012), 195-229; S. Peters and D. Westerståhl, Quantifiers in Language and Logic, (Oxford University Press, Oxford, 2006).

[13] There is also an Identity function in the set of duality relations $\mathcal{D}=\{\mathrm{ID}, \mathrm{E}, \mathrm{I}, \mathrm{D}\}$ which is not explicitly signified in the squares. In every vertex, however, there are two equivalent propositions, which implicitly show the Identity relation. If we also want to visually clearly indicate the Identity relation in each square, we just need to put a loop on all the vertices pointing to themselves.

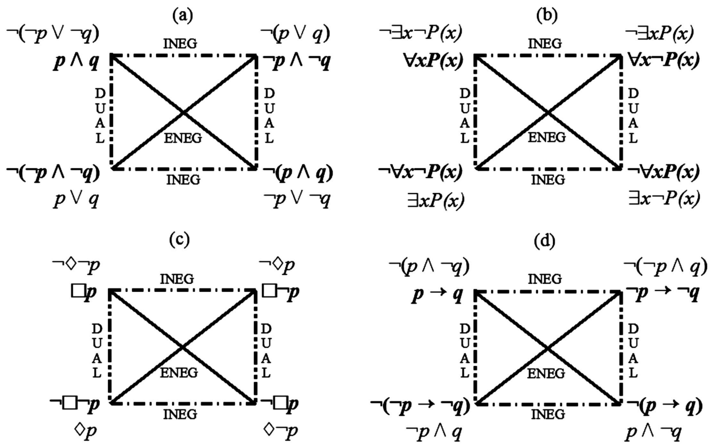

Figure 1　Duality Squares: (a) Conjunction and Disjunction (b) Universal quantifier and Existential quantifier (c) Necessity and Possibility operators (d) Material Implication

3　The Aristotelian relations

3.1　Informal characterizations

The typical Aristotelian logical relations appear firstly inDe Interpretatione. Aristotle says:

> I call an affirmation and a negation contradictory opposites when what one signifies universally the other signifies not universally, e. g. every man is white—not every man is white, no man is white—some man is white. But I call the universal affirmation and the universal negation contrary opposites, e. g. every man is just—no man is just. So these cannot be true together, but their opposites may both be true with respect to the same thing, e. g. not every man is white—some man is white. (17b. 16-26)[14]

In this passage Aristotle explicitly mentions two types of opposition relations which are contradictory and contrary. It amounts to say that *A* and *O* are contradictories, E and I are contradictories, and that A and E are contraries. Moreover, the contradiction relation constitutes the starting point of his treatment. This of course is consistent with his basic attitude towards the Law of Non-Contradiction (LNC). In his point of view LNC is the first principle which cannot be demonstrated. It is a primitive axiom of his entire logical system. As regards LNC, Aristotle actually give us several non-equivalent formulations. What are relevant here are the two versions of LNC distinguished by Łukasiewicz, which are ontological and logical respectively:

> It is impossible for the same thing to belong and not to belong at the same time

〔14〕 J. L. Ackrill, Aristotle: Categories and De Interpretatione, (Clarendon Press, Oxford, 1963).

to the same thing and in the same respect. (Met. 1005b19-23)

The opinion that opposite assertions are not simultaneously true is the firmest of all. (Met. 1011b13-14)

LNC imposes a requirement on the contradictory propositions, which is that they cannot be true together. But it does not mention the possibility about the distribution of the falsity. Therefore we are not able to get a pair of contradictions only by LNC. It is just not sufficient to distinguish between the contradictory opposition and contrary opposition. Contradiction needs a stronger logical property. This property is governed by another improvable logical law which is the Law of Excluded Middle (LEM):

Of any one subject, one thing must be either asserted or denied. (Met. 1011b24)

As a result, for any pair of contradictory propositions, one of them is true if and only if the other one is false. LNC guarantees them not to be both true and LEM guarantees them not to be both false.

After establishment of the logical properties of contradiction, we are now turning to investigating the contrary opposition. Based on the above quoted text fromDe Interpretatione, we can tell that Aristotle points out two conditions: firstly two contrary propositions cannot be true together; secondly their opposites (apparently meaning the contradictory opposites) can be true together. It is easy to infer from this that if the contradictories of the two contrary propositions can be true together, then according to the truth value distribution requirements of LNC those two contrary propositions can be false together.

Aristotle does not explicitly put forward the notion of subcontrariety. [15] In the passage we have seen, he only denotes them as the (contradictory) opposites of the contraries and states that they can be true together. Assuming that they are both false, the contradictories of them will both be true. They contradictories of them, however, are two contrary propositions, which cannot be true together. Therefore the propositions in the relation of subcontrariety cannot be false together. InPrior Analytics, Aristotle only counts the subcontrary propositions as verbal oppositions. Probably the reason is that for Aristotle there is a kind of strict sense for the notion of opposition to have, which is that for two propositions to stand in a mutually opposed relation they at least have to be mutually exclusive. So if Aristotle conceives the strict notion of opposition in terms of incompatibility, two propositions opposed to each other cannot be true together. It is not unreasonable, however, to

[15] Aristotle seems to have no technical term for the concept of subcontrariety. Normally speaking, he uses antiphasis and antiphatikos antikeimenos to name contradiction. Or as shown in the quoted text, antikeimenos is directly used alone to name contradiction. As for contrariety, he has the term enantiai. The first appearance of the notion of subcontrariety can be found in Apuleius' work Peri Hermeneias. In it, he uses the Latin term subpares for the concept of subcontrariety. Incongruae denotes opposition and alterutrae names contradiction. (M. W. Sullivan, Apuleian Logic, (North-Holland, Amsterdam, 1967), 65; Londey and Johanson, (1987), 56, 88-89, 111; Seuren. (2010), 152). The Greek counterpart of subcontrariae is hypenantiai the first use of which can be found in Ammonius' Greek Commentary on Aristotle's De Interpretatione: "The particulars are called sub-contraries, because they are placed below the contraries and follow from them." (A. Busse, Ammonius in Aristotelis De Interpretatione Commentarius, (Royal Prussian Academy of Science. Georg Reimer, Berlin, 1897), 92).

incorporate the subcontratiety in an indirect way into the traditional square of oppositions. After all, this concept can be ultimately defined by the concept of contrariety and the latter satisfies the condition for opposition in the strict sense.

The situation is similar in the case ofsubalternation. After the identification of the truth value distribution properties for contradiction, contrariety and subcontrariety, the relevant property of the subalternation relation naturally follows. As in the relation of subcontrariety, two propositions that are in subalternation can be true together. In light of Aristotle's choice of words, we may have to view them as opposed to each other only verbally. On the other hand, also like the case of subcontrariety, we are able to define subalternation in terms of contrariety. A and I are in subalternation only if there is an E such that A and E are contrary and I and E are contradictory. Another important thing worth noting is that subalternation involves directionality. That is to say, when A is true, I has to be true. The modal force here conveys the property of directionality and indicates the truth being transmitted from A to I, not the other way around.

In sum the typical four Aristotelian logical relations can be formulated informally as follows:

• Two propositions are contradictoryiff they cannot have the same truth value, i. e. cannot both be true and both be false.

• Two propositions are contrariesiff they cannot both be true but can both be false.

• Two propositions aresubcontraries iff they cannot both be false but can both be true.

• A proposition is a subaltern of anotheriff it must be true if its superaltern is true, and the superaltern must be false if the subaltern is false.

3.2 Formal definitions

In the course of trying to formally define the Aristotelian relations, it is necessary for us to pay special attentions to two of the features in our informal descriptions of them. Firstly, in our informal characterizations of contradiction, contrariety and subcontrariety, the word both occurs in all cases. It indicates one crucial criterion to distinguish among those three relations, which concerns the possibility of an identical truth value distribution. In the following formalizations, we will use the common sentential conjunction to deal with this criterion. Secondly there exists certain modal force in our informal definitions of the four Aristotelian relations, which is reflected in words like can and must. We consider this modal force expressing the notion of logical validity, which means we cannot define the Aristotelian relations simply by the logical connectives but have to define them in terms of valid and invalid relations.

Based on the above two points, we firstly define a simplified Aristotelian opposition language as $L(OP) = \langle P, C \rangle$. P is a countable set of predicates. C is the set of logical constants, including the necessary ordinary sentential connectives (here we take negation and conjunction as primitive) and the four special symbols for constructing the Aristotelian categorical propositions, which are a, e, i and o. In Aristotle's terminology, they correspond respectively to "hyparchein panti", "hyparchein oudeni", "hyparchein tini" and "ouch hyparchein tini". [16] The well-formed formulas of this language are pretty simple: if A and $B \in P$, then AaB、AeB、AiB and AoB are well-formed and based on those types of elementary propositions the negations and conjunctions of them can be formed. Further, let's

[16] See: J. Łukasiewicz, Aristotle's Syllogistic, (Clarendon, Oxford, 1957), 16.

assume a consequence relation ($\models$) for this language. Then for any two formulas φ and ψ in L, we define a set of Aristotelian relations $R = \{CD, CT, SCT, SA\}$ (corresponding respectively to contradiction, contrary, subcontrary and subalternation) as follows:

- CD(φ,ψ) iff $\models\sim(\varphi\wedge\psi)$ and $\models\sim(\sim\varphi\wedge\sim\psi)$;
- CT(φ,ψ) iff $\models\sim(\varphi\wedge\psi)$ and $\models\sim(\sim\varphi\wedge\sim\psi)$;
- SCT(φ,ψ) iff $\models\sim(\varphi\wedge\psi)$ and $\models\sim(\sim\varphi\wedge\sim\psi)$;
- SA (φ,ψ) iff $\models\varphi\rightarrow\psi$ and $\models\psi\rightarrow\varphi$.

$\models\sim(\sim\varphi\wedge\sim\psi)$ means that there exists a model capable of satisfying $\sim\varphi\wedge\sim\psi$. So the two propositions in contrary relations can be false together. Similarly $\models\psi\rightarrow\varphi$ means there is model capable of satisfying $\psi\wedge\sim\varphi$ which in turn determines $\psi\rightarrow\varphi$ to be invalid.

3.3 The Aristotelian opposition squares

It is not Aristotle himself that visually represents the four opposition relations defined as above by squares. The main focuses of Aristotle are on the relations of contradiction and contrariety, since only these two can be counted as true oppositions strictly speaking. But if just these two relations got represented diagrammatically, that would not bring square into existence. According toLondey and Johanson (1987), the earliest use of a square-like figure to present the four Aristotelian relations are done by Apuleius. As we already know, it is Apuleius who puts the subcontrary relation underneath, just parallel to the contrary relation. And it is him who proceeds to describe the situation in geometric terms: two universal propositions should be put on an upper line (superior linea) and two particulars should be placed on a lower line (inferiore linea) (Sullivan 1967). This kind of usage of words in describing the logical relations strongly suggests a square-like representation. Apuleius, however, has not yet identified the two subalternations, [17] which means that even if he actually drew a figure with four vertices it would not be a square but only a kind of crossed polygon. The square figure finally gets completed by the hands of Boethius. On the one hand, he explicitly puts forward the relation of subalternation, which provides the theoretical preparation for the possibility of adding two sides to Apuleius' polygon. On the other hand, he considers the visual presentation of the Aristotelian relations to be so important that he is self-consciously trying to do this. In his point of view, if the abstract objects of understanding could be concretely demonstrated in front of the eyes, they would be stored in our memories for a much longer period. [18]

In the following, we will present a few modern versions of the Aristotelian squares of oppositions:

[17] In fact, Apuleius does not have any terminology for subalternation.

[18] Superioris autem disputationis integrum descripionis subdidimus exemplar, quatenus quod animo cogitationique conceptum est oculis expositum memoriae tenacious infigatur. (Meiser 1880: p. 152)

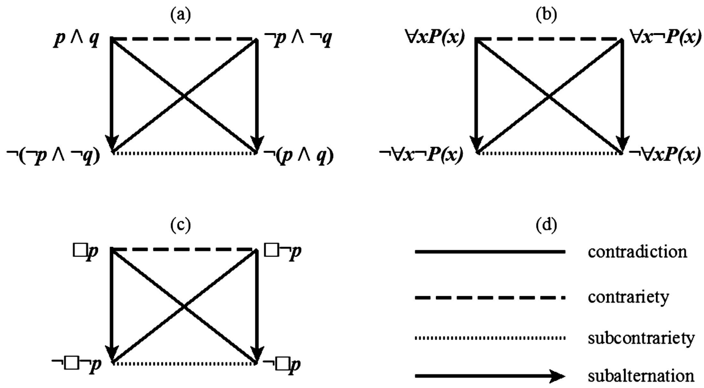

Figure 2　Aristotelian square of oppositions: (a) conjunction (b) universal quantifier (c) necessity operator (d) codes

4　The seminaries and dissimilarities of the duality squares and the Aristotelian squares

4.1　Structural similarities

By comparison of figure 1 and 2, it is easy to see that the squares constructed by the connectives, quantifiers and modal operators are both duality squares and Aristotelian squares. On the diagrammatical level, there seems to be some kind of isomorphism between those duality squares and opposition squares. In any given duality square, we have 2 external negations, 2 internal negations and 2 duals. In any given opposition square, we have 2 contradictions, 1 contrariety, 1 subcontrariety and 2subalternations. Since external negation is the most common and typical way to construct a pair of contradictories, we tend to directly associate the 2 contradiction relations in an opposition square with the 2 external negation relations in a duality square. This parallelism is well-established and very neat.

At the same time, it is known that in propositional logic the formula $\varphi \to \psi$ is equivalent to the formula $\sim(\varphi \wedge \sim\psi)$. And given any two operators O and Q, if they are each other's dual, then $O \equiv \sim Q \sim$. Therefore, it gives us an apparent impression that both in the case of subalternation and dual, there involve the interactions between external and internal negation. Of course, the impression is misleading to the effect that the so-called internal negation in the formula $\sim(\varphi \wedge \sim\psi)$ is a negation only applied to ψ. Besides that, the most common arrangements of the diagrams themselves also strongly induce us to make this correspondence between subalternation and dual, as they are both drawn as occupying the two sides of a square. After establishing these two isomorphisms, the third one naturally follows, which is between internal negation on one side and contrariety and subcontrariety on the other. The problem here, however, is that the corresponding between internal negation and contrariety and subcontrariety is not perfect. It is not a 1-1 mapping, since there are two Aristotelian relations which cause the mess of 1-2.

Generally speaking, if the attention were limited only to the cases of sets of four propositions each of which has only one dominant operator, significant structural similarities between the duality squares and the Aristotelian squares would very likely be found, even to the point of isomorphism.

No matter the square is constructed from conjunction-disjunction pair or from quantifiers or from modal operators, it is both a duality square and an Aristotelian opposition square. And this kind of overlapping seems to result from the interaction between external negation and internal negation. It is obvious and intuitive. With respect to the duality squares, the relations embodied in them are just defined by external and internal negation. As for the Aristotelian relations, the reasons leading to the different situations of truth value distributions are also tended to be thought as consisted in the different positions of the occurrence of negation.

It is true that there is a pretty tight connection between the Aristotelian relations and the negation operations. This fact has already been noticed and confirmed by the medieval logicians. In Peter of Spain's work, he proposes three rules forequipollences (SL. Ⅰ18):

• If before any sign we put a negation, it is equipollent to its contradictory.

• If after any universal sign we put a negation, it is equipollent to its contrary.

• If before and after any universal or particular sign we put a negation, it is equipollent to itssubalternate.[19]

William of Sherwood also puts forward similar rules and further concludes them by the help of a mnemonic verse as follows: Pre contradict, post contrariatur, pre postque subalternantur. (Introductiones 19)[20]

The tight connection between the Aristotelian relations and the duality relations, also between the corresponding diagrams, is significantly instantiated by the association of the contradiction of the external negation. In his comment on the above-mentioned first rule of equipollence, Buridan points out that: "there is no better and more reliable way to form the contradictory of a proposition than by prefixing a negation to it that is understood to be operating on the whole proposition." (Summulae 1.5.2)[21]

From the diagrammatic point of view, inspection and comparison of some of the so called 'collapsed' duality squares and 'degenerate' opposition squares help to shed some lights on the nature of the close connection between contradiction and external negation, which in its turn is conductive to dealing with the problem of similarity between those two sets of squares.

The most famous degenerate version of the traditional square of opposition is its modern

[19] De equipollentiis assignantur regule tales: si alicui signo preponatur negatio, equipollet suo contradictorio. Et ideo equipollent iste: non omnis homo currit/quidam homo non currit; et ita de aliis. Secunda regula talis est: si alicui signo universali postponatur negatio, equipollet suo contrario, sicut iste: omnis homo non est animal/nullus homo est animal; vel iste: nullus homo non currit/omnis homo currit; et ita de aliis signis universalibus affirmativis et negativis. Tertia regula est talis: si alicui signo universali vel particulari preponatur et postponatur negatio, equipollet suo subaltern. sicut iste: non omnis homo non currit/quidam homo currit; et iste similiter: non quidam homo non currit/omnis homo currit. Et sic de quolibet alio signo. (B. Copenhaver, C. Normore and T. Parsons, Peter of Spain: Summaries of Logic. Text, Translation, Introduction, and Notes, (Oxford University Press, Oxford 2014), 116-117).

[20] See: N. Kretzmann, William of Sherwood's Introduction to Logic, (University of Minnesota Press, Minneapolis, 1966), 38. Peter of Spain also likes to propose such kinds of mnemonic verses. Actually, there is the conjecture that one of the reasons that the logical works of Peter of Spain are more popular at that time than the ones of William of Sherwood is simply that the formers contain much more and better mnemonic verses (Kneale, The Development of Logic, (Clarendon Press, Oxford, 1962), 234). And generally speaking, the way of using metrical devices to help understanding and memory begins to be popular at least since the twelfth century.

[21] Et ista regula sic correcta est universaliter vera, sive in propositionibus categoricis sive in hypotheticis, sive in propositione sit aliquod signum sive non, quia non potest melius et firmius sumi contradictio ad aliquam propositionem quam sibi praeponendo negationem quae intelligatur cadere super totam propositionem. (Hubien 2001)

revision. As we know, a necessary condition to assure the validity of the Aristotelian relations is the existential import of the affirmative propositions. Specifically, in the Aristotelian logical system, not only do the particular affirmatives have existential import, but also the universal affirmatives do. That is to say, in Aristotelian logic, a typeA proposition as "Every S is P" should be formalized as $x\ (Sx \to Px) \wedge \exists xSx$. Therefore, if there is no entity having the property expressed by S in our domain, the universal judgment is simply false. Only by the warrant of truth conditions determined by the existential import, the logical relations presented by the Aristotelian square of opposition are valid. In contemporary classical first-order logic, however, universal affirmatives do not have existential import. If we formulate the four quantified propositions into the notations of standard first-order predicate calculus, we would end up losing most of the logical relations embodied in it. For example, if we symbolize the logical form of 'Every S is P', 'No S is P' and 'Some S is P' respectively as $\forall x(Sx \to Px)$, $\forall x(Sx \to \sim Px)$ and $\exists x(Sx \wedge Px)$, it is not difficult to see that according to the standard truth conditions of these quantified formulas A and E can both be true if the contained subformula Sx is false, especially when there is no x such that Sx. For the same reason, A doesn't imply I in the sense that there is a model in which A is trivially true but I is false. Therefore, without existential import the traditional square of opposition would be doomed to be a degenerate one, which would give rise to the following diagram with only two pairs of contradictions as survivals:

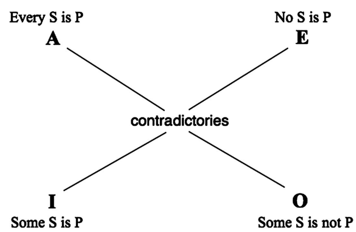

Figure 3　a degenerate Square of Oppositions

As for the duality squares, the possibility of collapse comes from the loss of efficacy of the negation operation. That is to say, for any given operator, theoretically speaking, applying any operation in the set $D=\{\mathrm{ID}, E, I, D\}$ gives us the original operator itself. It will be shown soon, however, that in reality the efficacy of E cannot be lost in order to secure the consistency of the underlying logical system. If the efficacy of D is cancelled, then for any operator O, $D\ (O) \equiv O$. O becomes the dual of itself. Since $E \circ D(O) = I(O)$, by substitution of equivalents, it follows that E(O)=I(O). For a self-dual operator, its external negation equals its internal negation. Under such a circumstance, the duality square collapses into the diagram below:

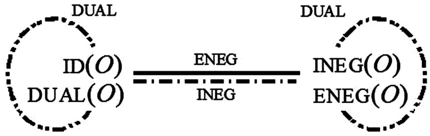

Figure 4　A collapsed square constructed by a self-dual operator

Similar possibility also occurs to internal negation operation. If the efficacy of I is cancelled, then for any operator O, $I(O) \equiv O$. Since $D \circ I(O) = E(O)$, by substitution of equivalents, it follows that

$D(O)=E(O)$: for an operator that is the internal negation of itself, its dual equals its external negation. Under such a circumstance, the duality square also collapses:

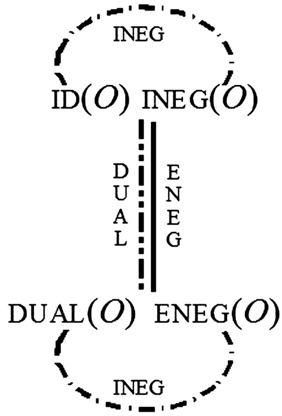

Figure 5 A collapsed duality square constructed by an operator which is its own internal negation

Taking the collapsed situations above into account, it is easy to discover that the duality square and the Aristotelian square have the similar pattern of collapse. In the two cases of collapsed duality, only the relation of external negation is substantially preserved. If the supposed structural isomorphismwere to map external negation to contradiction, we would be in a position to expect that in the case of degenerate opposition the only survivor would be contradiction, which is exactly what happens. And if we move further to consider the possibility for E to collapse, which is amount to identify an operator with its own external negation, we would be led to directly violate the Law of Non-Contradiction. In such a situation, the result of applying the operator in question to any operand would be true and false simultaneously. In order to guarantee the validity of LNC, the application domain of such an operator has to be empty which automatically cancels the efficacies of E, I, and D altogether. The whole square would be reduced to a point.

In light of all considerations above, we are strongly inclined to think that the Aristotelian logical relations and the duality relations are conceptually interdependent and even equivalent. The isomorphism exhibited at the level of squares seems to result from their characterizing essentially the same set of logical relations. Extensionally speaking, nearly all the frequently discussed Aristotelian squares are at the same time duality squares, and vice versa. In the following section, however, efforts are made to show that this first impression based on simple squares is not reliable and highly misleading. In fact, the Aristotelian relations and the duality relations are different in nature and conceptually independent. The satisfaction of the requirements imposed by one set of relations is not sufficient and necessary for the satisfaction of the other.

Now, away from the abstract treatment of dissimilarities between the Aristotelian relations and duality relations, the discussion is moving to a concrete diagram proposed by Buridan. It is not a simple square containing 4 propositions but a very complicated octagon. As will be seen soon, when more propositions and more operators are taken into account, the resulting diagrams will be more complicated and the seeming isomorphism diagrammatically revealed before at the simple geometric level will be largely lost.

4.2 The differences between the Aristotelian relations and the duality relations

Investigating the logical properties of the two sets of relations, it is not difficult to discover some apparent differences. Firstly, from the cases of the two collapsed duality squares discussed above, we notice that at least D and I can be reflexive. In Aristotelian relations, however, there is no reflexive one. One of the consequences followed from reflexivity is that with respect to a self-dual operator there exist two propositions standing both in the relation of external negation and the relation of internal negation. By the same reason, with respect to an operator that is its own internal negation, there exit two propositions standing both in the relation of external negation and the relation of dual. No two propositions, however, can stand in more than one Aristotelian relation. The Aristotelian relations are mutually exclusive.

Secondly and most frequently pointed out by various scholars, the duality square is perfectly symmetrical. For any relation R in $D=\{\mathrm{ID}, E, I, D\}$, ORQ iff QRO. In Aristotelian relations, however, not all the relations are symmetric. Subalternation is just one-sided.

Finally, although any two arbitrary propositions can stand in only one Aristotelian relation, one proposition is allowed to be in relations with several non-equivalent propositions with respect to the Aristotelian relations except for contradiction. That is to say, although one proposition has only one contradictory counterpart, it is capable of having more than one contrary, subcontrary and subalternate counterparts which are all not equivalent. As for the duality relations, however, any proposition can have only one external, internal negation and only one dual. They are all functions, which is also the ground for us to form the set $O(Q)=\{\mathrm{ID}(Q), E(Q), I(Q), D(Q)\}$ based on $D=\{ID, E, I, D\}$.

Besides the above apparent dissimilarities of the logical properties between the two sets of relations, their definitions reveal some deeper differences. Firstly, the definitions of the duality relations essentially involve the interaction between the external and internal negation. They inevitably impose some restrictions of different levels of strength on the domain of duality. The specific strength of the restrictions depends in large part on how to exactly understand the notion of negation in question. If we confine the notion of negation within a syntactic level, the restrictions become the strongest, since it requires that for any dual relation a grammatical negation is possible. There exist, however, various kinds of predicates the grammatical negations of which are ungrammatical themselves but which still seem to be able to stand in the duality relations. Therefore, it would be better for us to understand negation semantically, which means that the indication of the presence of negation is not the necessary occurrence of certain grammatical negation signs. We need to appeal to some semantic criteria, which would consequently enlarge the domain of duality.

Even so, there is still a potential misleading element in the definitions of duality which concerns the notion of an operator. It is not clear how we should understand this notion precisely. In the most natural way, the typical examples of an operator are given to us as the sentential connectives, the quantifiers and the modal operators. They all take one or more predicate type operands that can allow negation imposed upon them, which realize the internal negation in the definition. Furthermore, the combination of those operators with their operands can also be negated, which realize the external negation. With these two requirements fulfilled the duality relations can be well-defined. Therefore, construed in this way, any duality operators, that is any element of the set $O(Q) = \{ID(Q), E(Q), I(Q), D(Q)\}$, has to be at least second-order. According to the definition and the common notation of type in formal semantics, the sentential connectives are of the type $\langle t, t\rangle$ and the quantifiers are of the type $\langle\langle\langle e, t\rangle, e\rangle, t\rangle$. If all the elements in $O(Q)$ are at least second-order, then the duality relations between them, that is all the elements in $D = \{\mathrm{ID}, E, I, D\}$ has to be at least third-order.

Secondly, the Aristotelian relations are explicitly defined along the semantic dimension, which directly appeals to the different distributions of truth values without stipulatinghow to achieve those distributions syntactically or semantically. The comment made by Buridan on Peter of Spain's first rule of equipollence has already been mentioned above. In his view, external negation is the best and most reliable way to construct contradictories. But logically speaking, there may still be other ways to form a pair of contradictories. In his comment on the second rule of Peter of Spain, Buridan states that since the negation does not precede the universal sign, it does not remove its universality, but, since it precedes the copula, it changes the quality of the proposition; therefore, those two both remain universal, the one being affirmative and the other negative; and such are contraries (Summulae 1. 5. 3). [22] Similarly in his comment on the third rule, Buridan says that since two negations occur before the copula, they cancel each other such that the quality of the proposition stays the same; But the negation before the universal sign changes the quantity such that the two propositions are in subalternation (Summulae 1. 5. 4) [23]

It is clear fromBuridan's comments that in discussing the interactions between negations and the results thus produced he actually presupposes the Aristotelian relations. The latter get defined and generated independently, although the negation operations can have the same generative effects in a certain sense. On the one hand, not all instances of the Aristotelian relations are created by the operation of negation. On the other hand, not all manipulations of negation would produce an instance of an Aristotelian relation. The operation of negation and the generation of Aristotelian relations are conceptually completely independent. They are basically two distinct processes; just sometimes in some case converge to some extent. Buridan's comments actually suggest a methodology to compare the Aristotelian relations and the duality relations: presupposing one of the two sets of logical relations on independent grounds and then going on to see if the operations from the other set can be used to produce the presupposed relations.

Although in the formal definitions from section 3. 2 the domain of the Aristotelian relations is limited to the well-formed formulas ofL(OP), it actually can be generalized and defined over any first-order predicate as internal negation is out of question. Therefore, different from the duality relations as stipulated in the syntactic definitions (not necessarily in the group-theoretic approach),

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〔22〕 Causa huius regulae est quia negatio, cum non praecedat signum, non removet eius universalitatem, sed, quia praecedit copulam, mutat qualitatem propositionis; ideo illae remanent ambae universales, una affirmativa et alia negativa, et tales sunt contrariae. (Hubien, Johannes Buridanus: Summulae de dialectica. http://www. logicmuseum. com/wiki/Authors/Buridan/Summulae _ de _ dialectica, 2001).

〔23〕 Et causa huius regulae est quia illae duae negationes praecedentes copulam destruunt se in ordine ad copulam, ideo dimittunt qualitatem propositionis eandem. Istae enim ambae sunt affirmativae 'omnis homo currit' et 'non omnis homo non currit', et istae ambae negativae 'nullus homo currit' et 'non nullus homo non currit'. Sed quia sola negatio praecedit signum, ideo mutat quantitatem propositionis. Modo tales sunt subalternae, scilicet eiusdem qualitatis et diversae quantitatis, una universalis et alia particularis. (Hubien, 2001) any Aristotelian relation can be a second-order relation holding between two first-order predicates. For example, let's consider the following 4 first-order predicates: "is a cat", "is a dog", "not is a dog" and "not is a cat". The Aristotelian relations are able to be defined legitimately on them. "is a cat" and "is a dog" cannot be true of the same argument, but can be false of the same argument, which makes them contraries. "is a cat" and "not is a cat" are contradictory to each other, so are "is a dog" and "not is a dog". The final two subalternation relations are easily established. The 4 predicates in question, however, have no way to form a duality square, for there is no spot for internal negation.

5 Buridan's modal octagon

After the abstract discussion of the dissimilarities between the Aristotelian relations and duality relations, it's time to go back to a concrete diagram. If the arguments proposed above were accepted, it would be expected that there should be a certain type of diagram which is able to dispel the isomorphism illusion created by a simple square and further clearly shows the differences between opposition relations and duality relations. Without any doubt, Buridan's modal octagon provides us with a perfect example.

The advantage ofBuridan's octagon lies in the fact that it is both an extension of the traditional square of oppositions and of a square of dualities. Since the sets of relations are simultaneously presented in one and the same square, we are in a position to observe that the correspondence situation between them is actually quite disordered, so far from being an isomorphism.

Buridan's modal octagon is an extension of the Aristotelian square for the reason that it is essentially a combination of two basic opposition squares, i. e. the square created by quantifiers and the square created by modal operators.

With respect to a traditional opposition square constituted by quantifiers, Buridan makes the relevant enlargement as such: instead of putting one proposition at one vertex, he substitutes it by a set of equivalent propositions. For example, at the A corner of the traditional square, he puts 6 propositions, 3 of which are logically relevant for the square:

'every man is running' (omnis homo currit).
'no man is not running' (nullus homo non currit)
'not any man is not running' (non quidam homo non currit)

The three propositions are equivalent, which results from the duality among the operators in them. The first one concerns the quantifier ∀. The third one concerns the quantifier ∃ which is lexicalized in English in this context as any and which is the dual of ∀. The second one concerns the external negation of ∃ which is lexicalized as no and which is the internal negation of ∀. This arrangement of propositions clearly shows that the duality properties of operators are well understood by Buridan. Theoretically speaking, it is anticipated that a traditional modal square should also be expanded by Buridan through putting 3 propositions at each vertex. This expectation is assured in Dorp's Compendium. There actually exists a modal square the vertices of which contain 3 equivalent propositions. Combining the two squares together, we get a polygon each vertex of which has 9 equivalent propositions. Furthermore, combining the quantifiers and the modal operators, Buridan produces the following 8 propositions (Summulae 1.8.6):

1) all B are necessarily A

2) all B are possibly A

3) some B are necessarily A

4) some B are possibly A

5) all B are necessarily not A

6) all B are possibly not A

7) some B are necessarily not A

8) some B are possibly not A

These 8 propositions make sure that the new polygon be an octagon. Each vertex of it contains one of the propositions above plus another 8 propositions that are equivalent to it. And all these together constitute the basic frame ofBuridan's modal octagon.

The above 8 propositions embody 8 possible interactions between the quantifiers and modal operators, which are 1. ∀□; 2. ∀ ; 3. ∃□; 4. ∃ ; 5. ∀□∼; 6. ∀ ∼; 7. ∃□∼ and 8. ∃ ∼. Two pairs of operators are concerned with here. According to duality, however, the situation can be reduced to involve only two operators. Since ∃≡∼∀∼ and ≡∼□∼, by substitution, we get: 1. ∀□; 2. ∀∼□∼; 3. ∼∀∼□; 4. ∼∀□∼; 5. ∀□∼; 6. ∀∼□; 7. ∼∀∼□∼ and 8. ∼∀□.

The 8 combinations exactly represent the integration of a quantifier duality square and a modal duality square. Each square refers to one operator and the 4 distribution possibilities of its negation. The result of combining two operators is the addition of the third position for negation. Besides the original external and internal negation, we acquire anothernegation possibility just in the middle of the two operators, which accordingly constitutes the reason why the Buridan's Octagon is an extension of a duality square.

Based on those 8 modal propositions, Buridan works out the 28 combination possibilities between any two different propositions of them (Summulae 1.8.6). Furthermore, Buridan investigates each one of the combinations in order to see if it satisfies any Aristotelian relation or not. At last, he reaches the following conclusions: there are 10 subalternations, 5 contrarieties, 5 subcontrarieties, 4 contradictions and 4 disparate relations (standing in no Aristotelian relations at all) (Summulae 1.8.6).[24] On account of all the theoretical studies, Buridan presents his modal octagons, one simplified version of which looks like the following figure, which is mainly presented from the first viewpoint:

[24] Decem sunt subalternationes, quinque contrarietates, quinque subcontrarietates, quattuor contradictiones et quattuor disparationes. (Hubien 2001)

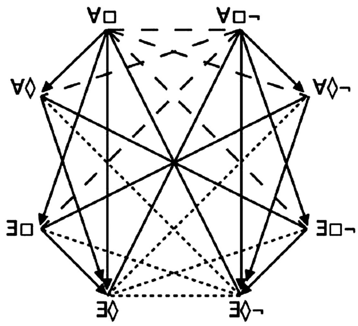

Figure 6　Buridan's modal octagon

If we want to examine this octagon clearly from the viewpoint of duality, it is only required to make the relevant substitutions, which highlights the distribution pattern of negations. Besides the old combinations, we have external + inter negation (EI), external + middle negation (EM), middle + internal negation (MI) and external + middle + internal negation (EMI):

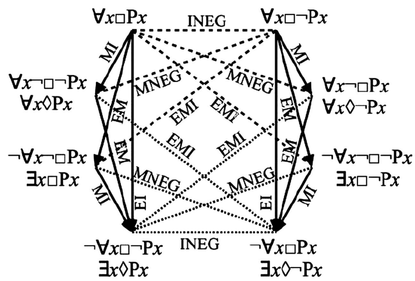

Figure 7　Buridan's modal octagon from the duality point of view

From Buridan's complex combinations of operators and negations, it can be discovered that between the Aristotelian relations and the duality relations only contradiction and external negation stand in one to one correspondence. As a matter of fact, the 4 contradiction relations correspond just to the 4 external negation relations. Different from the structural properties exhibited on the level of squares, the results of internal negation are not only contrariety and subcontrariety, but also 2 disparate relations. And the results of external negation of internal negation do not include just subalternations, but also 2 other disparate relations. By parity of reasoning, all the combination possibilities can be established. Diagrammatically speaking then, it is pretty clear that the correspondence between the Aristotelian relations and the duality relations are so chaotic that besides contradiction and external negation it is impossible to propose any equivalent principle associating an Aristotelian relation with a negation operation as the any one suggested by Peter of Spain .

6　Conclusions

The Aristotelian relations and the duality relations are two independent sets of logical relations. Apart from a variety of apparent differences between their logical behaviors, there are essential dissimilaritieson the level of concept. The formers are given explicit semantic definitions in terms of the truth conditions of propositions or the satisfaction conditions of any predicate. The specific syntactic or semantic way to achieve a certain truth value distribution is not stipulated. Hence the Aristotelian relations can be defined on any first order predicate and express second order relations between them.

The duality relations concerned with in this article, however, are defined on the interactions between negations. In the simple case of single operator, there are 2 negation spots and 4 combination possibilities of external and internal negations. InBuridan's modal octagon, there are 2 operators, 3 negation spots and 8 distributions of negations. Since duality relations require the operation of negation, the operator itself has to be of predicate type and at least one of the operands has to be of predicate type. Therefore, any duality operator has to be at least of second order and the relation between any two of them has to be at least of third order.

On the diagrammatic level, simple squares involving just 4 propositions are very misleading in the sense that opposition squares and duality squares are seem to be isomorphism based on the fact that a good deal of interesting Aristotelian squares turns out to be duality squares at the same time and vice versa. When we turn to more complicated diagrams, such asBuridan's modal octagon, the huge differences between them are made prominent. The octagon considered, both as an extension of a duality square and an opposition square, presents more instances of the two relations. By close comparison of all the instances, the correspondence between the Aristotelian relations and the duality relations are shown to be so chaotic that besides contradiction and external negation it is impossible to propose any equivalent principle associating one of the former with one of the latter.

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中文题目:

亚里士多德式逻辑关系与对偶性关系的异同:从传统正方形到布里丹八边形

马明宇,比利时鲁文天主教大学哲学系在读博士生,Tel:+8613984053530,Email:mamingyuleon@163.com

摘要:本文将处理两套逻辑关系:"亚里士多德式关系(Aristotelian relations)"和"对偶性关系(Duality relations)"。对这两套关系进行图形化表达,可以得到亚里士多德式对立正方形和对偶性正方形,它们似乎具有某种同构关系。大量有趣的亚里士多德式正方形同时也是对偶正方形,反之亦然。然而,本文试图阐明,这两组逻辑关系既不等价,也不同构。它们不仅拥有众多逻辑特征上的差异,从本质上和概念上来说,也是相互独立的。随着我们加入更多的公式,展现这些公式间相互逻辑关系的图形就将变得更加复杂,二者间的差异也就更加明显。本文所选的复杂图形,是由14世纪逻辑学家布里丹所提出的八边形。我们将会看到,这个图形既是对立正方形的扩展,也是对偶正方形的扩展,其中所体现的亚里士多德式关系和对偶关系具有明显的非同构性。

关键词:亚里士多德式关系;对偶性关系;传统对立正方形;对偶性正方形;布里丹八边形