A Binary Approach to Modeling Complex Urban Systems

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Abstract: Cities are nonlinear, complex systems that defy traditional analytical models to depict. New modeling techniques are needed that help us to understand the underlying mechanisms of how complex urban systems work. In this paper, we present two binary systems: the I Ching and the elementary cellular automata and show how they relate to each other. In particular, we attempt to use this binary model in combination with spatial games to explain how mixed use in land comes about. Insights into how cities work can be gained from such modeling techniques.

Key Words: I-Ching; Cellular Automata; Binary Systems; Complexity Sciences; Cities

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1 Introduction

Traditional theories, such as decision theory, that aim at linearity and simplicity are insufficient in dealing with the world that is nonlinear and complex. Recently, a complexity science was being built aiming at the understanding of the complex world across the traditional boundaries of natural and social sciences, which triggers off a search for a science of cities[1]. The main distinction between the complexity science and the classical science is emergentism vs. reductionism in that the former views natural and social laws as principles of self-organization emerging internally from the systems under consideration, rather than imposed externally as presumed by the latter. In this paper, we argue that the traditional Eastern thoughts are inherently emergent, including Buddhism and Taoism, which view the world as transient phenomena and are partly consistent with the philosophy of the complexity science. Given the rise of complexity movement[2], we argue that the time is ripe to integrate Eastern and Western lines of thought and sciences into a coherent framework in order to understand how the complex world works and how we can cope with it. In particular, we will propose a new way of integrating the I Ching, the Book of Change, with the elementary cellular automata discovered recently by Wolfram[3]. Specifically, the 256 transition rules of the elementary cellular could be used to interpret the meanings of the 64 hexagrams in the I Ching which, in our view,

[1] Batty, Michael. 2013. *The New Science of Cities*. Cambridge, MA: The MIT Press.

[2] Arthur, W. Brian. 2015. *Complexity and the Economy*. Cambridge, UK: Oxford University Press.

[3] Wolfram, Stehpen. 2002. *A New Kind of Science*. Champaign, IL: Wolfram Media.

remain mythical. Each transition rule in the elementary cellular automata corresponds to eight relationships between a triple element and a value of zero or one, or yin and yang in the I Ching, while each hexagram in the I Ching has six elements which form a subset of each transition rule. Thus, each hexagram in the I Ching corresponds to four transition rules in the elementary cellular automata, forming the possible states of each hexagram or phenomenon in the universe.

Cities are now recognized as complex systems. We have shown elsewhere how the elementary cellular automata could be used to model cities in that we could treat the values of the cells, that is zero/one or yin/yang, as residential/commercial land uses or buildings/activities[4]. We have also shown elsewhere that we could use the elementary automata to investigate the effects of planning resulting in the system's orderly behavior[5]. Coupled with genetic algorithm, the binary systems could teach us how to plan to achieve what was intended to[6]. All this could be linked to the I Ching as a way of looking at how cities work and what plans could achieve. As one of the earliest complexity theories, the I Ching is arguably the fundamental philosophy of planning related activities in East Asia, including China, Japan, Korea, and Taiwan, so it is worth teaching the philosophy in the light of modern, western science in a planning curriculum. Such integration would enrich our approaches to understanding and dealing with cities.

The notion that the world we find ourselves is complex is not a new idea. However, not until recently has complexity been seriously studied among academics. In other words, complexity in our world has long been recognized, but largely ignored in scholarly work. It is particularly so in the field of urban planning in that the notion of urban complexity was recognized not until the 1960s and only recently has serious search for a science of cities begun[7]. As a result, we are still short of technology for dealing with the complex world mainly because the technology derived from modern western sciences of reductionism is claimed to solve simple problems. We would argue that with the rise of emergentism in complexity science, which the traditional Eastern philosophies seem to be based on, it is possible to bridge the gap between Western modern sciences and Eastern traditional philosophies to explore into possible ways of understanding and dealing with the complex world in which we find ourselves.

In the present paper, we will introduce two binary systems of complexity from an Eastern traditional philosophy and a Western modern science respectively, namely the I Ching and the elementary cellular automata discovered by Wolfram[8], followed by a depiction of how these two binary systems can be incorporated into a hybrid binary system of complexity. In Section 3, we will demonstrate how the resulting model can be applied to the understanding of urban spatial change. In Section 4, we will discuss some limitations of the model. In the concluding section, we will argue that this integration of Eastern and Western lines of thought would not only enrich our scholarly work in understanding how the world works by linking the two seemingly separate, large literatures, but also

[4] Lai, Shih-Kung. 2003. "On transition rules of complex structure in one-dimensional cellular automata: Some implications for urban change," *Annals of Regional Science*, 37(2): 337-352.

[5] Lai, Shih-Kung and Haoying Han. 2014. *Urban Complexity and Planning: Theories and Computer Simulations*. London: Ashgate.

[6] Mitchell, Melanie. (2009). *Complexity: A Guided Tour. Cambridge*, UK: Oxford University Press.

[7] Batty, Michael. 2013. *The New Science of Cities*. Cambridge, MA: The MIT Press.

[8] Wolfram, Stehpen. 2002. *A New Kind of Science*. Champaign, IL: Wolfram Media.

provide a fresh look at cities recognized as complex spatial systems to improve our capabilities of dealing with urban complexity.

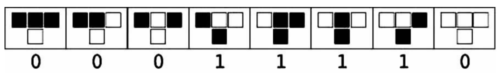
2 The Model

2.1 Binary System 1: The I Ching

There are 64 guas in Binary System 1, or the I Ching, each of which is composed of six yaos. Each yao can be either yin or yang, or zero or one in binary terms. For example, the Qian (乾) gua has six yangs, or 111111, whereas the Kun (坤) gua has six yins, or 000000. The 64 guas can be represented in terms of a binary system as in Table 1.

2.2 Binary System 2: The Elementary Cellular Automata

In the elementary cellular automata[9], there are 256 transition rules, each of which is composed of eight triplets. For example, Rule 30 has the following representation:

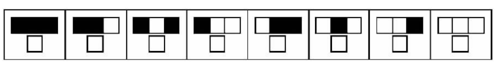


Note a black cell has a value of one, while a white cell contains a value of zero and can be coded as 00011110. All other transition rules can be constructed in a similar way.

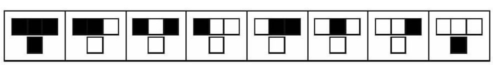
2.3 Binary System 3: Complex I

There are totally 64guas in the I Ching, while there are totally 256 transition rules in the elementary cellular automata. One way to relate guas to transition rules is to take the six central binary digits in a transition rule corresponding to a particular gua, so that each gua corresponds to four transition rules. For example, Kun (坤) gua corresponds to transition rules 0, 129, 1, and 128 as follows.

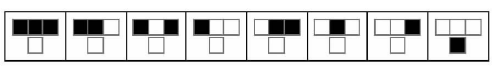
Rule 0



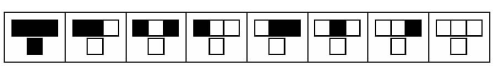
Rule 129



Rule 1



Rule 128



[9] Wolfram, Stehpen. 2002. *A New Kind of Science*. Champaign, IL: Wolfram Media.

Table 1 The Binary System of the I Ching

Qian	(乾)	111111	Dun	(遯)	001111
Kun	(坤)	000000	Dazhuang	(大壯)	111100
Tun	(屯)	100010	Jin	(晉)	000101
Meng	(蒙)	010001	Mingyi	(明夷)	101000
Xu	(需)	111010	Jiaren	(家人)	101011
Song	(訟)	010111	Kui	(睽)	110101
Shi	(師)	010000	Jian	(蹇)	001010
Bi	(比)	000010	Jie	(解)	010100
Xiaoxu	(小畜)	111011	Sun	(損)	110001
Lu	(履)	110111	Yi	(益)	100011
Tai	(泰)	111000	Guai	(夬)	111110
Pi	(否)	000111	Gou	(姤)	011111
Tonghren	(同人)	101111	Cui	(萃)	000110
Dayou	(大有)	111101	Sheng	(升)	011000
Qian	(謙)	001000	Kun	(困)	010110
Yu	(豫)	000100	Jing	(井)	011010
Sui	(隨)	100110	Ge	(革)	101110
Gu	(蠱)	011001	Ding	(鼎)	011101
Lin	(臨)	110000	Zhen	(震)	100100
Guan	(觀)	000011	Gen	(艮)	001001
Shihe	(噬嗑)	100101	Jian	(漸)	001011
Bi	(賁)	101001	Guimei	(歸妹)	110100
Bo	(剝)	000001	Feng	(豐)	101100
Fu	(復)	100000	Lu	(旅)	001101
Wuwang	(無妄)	100111	Xun	(巽)	011011
Daxu	(大畜)	111001	Dui	(兌)	110110
Yi	(頤)	100001	Huan	(渙)	010011
Daguo	(大過)	011110	Jie	(節)	110010
Kan	(坎)	010010	Zhongfu	(中孚)	110011
Li	(離)	101101	Xiaoguo	(小過)	001100
Xian	(咸)	001110	Jiji	(既濟)	101010
Heng	(恆)	011100	Weiji	(未濟)	010101

3 Applications

The game——theoretic analysis depicted in the present paper can be used to draw implications in relation to urban development. For example, elsewhere we argued that mixed use in land may occur regardless of whether a zoning system is imposed on a city, due to the sensitivity of spatial pattern to mixed use in the zoning system[10]. Based on the formulation to be depicted shortly, it can be further

〔10〕 Lai, Shih-Kung and Haoying Han. 2012. "On failure of zoning," *International Journal of Society Systems Science*, 4(4):369-380.

argued here that mixed use yields higher social welfare than single use.

Consider two land uses, commercial and residential, each of which could be adopted by two developers on two neighboring parcels in a community. This case can be easily interpreted as a spatial game of repeated prisoner's dilemma[11][12]. Assume further that the developer could adopt one of the four strategies: TFT (tit-for-tat), AD (always defect), AC (always cooperate), and RA (random action). In a TFT strategy, the player cooperates initially and then follows the action taken by the other player in the ensuing encounters. In the AD or AC strategy, the player always defects or cooperates, regardless of what the other player does. In the RA strategy, the players defects or cooperates randomly.

We have proved that TFT outperforms descriptively and normatively the other three strategies in terms of social welfare. Apparently, in a zoning system, only AD or AC can be adopted, assuming that defection and cooperation means commercial and residential use respectively. The resulting spatial pattern would be either commercial or residential use, to conform to the zoning ordinance. Such a strategy is, however, suboptimal because neither one yields higher social welfare than TFT. In a permit system that allows mixed use, the developer would be motivated to adopt the TFT strategy, which is optimal and yields the highest payoff across the players among all the strategies. The resulting spatial pattern is likely to be mixed use. The implication is that the zoning system of single use might perform less effective than the permit system of mixed use in terms of social welfare, other things being equal.

To simplify, consider an array of cells. In the land development context, suppose that each cell represents a block and there are only two permissible types of land use for each block, residential or commercial. The payoff matrix for the land development is written as follows:

	R	C
R	1	0
C	b	0

Note that R stands for residential and C symbolizes commercial with b as the parameter in the original payoff table of the two-person, iterated prisoner's dilemma game[13]. Whether single or mixed use pattern would prevail is determined by the parameter b. This question can be investigated through one-dimensional elementary cellular automata[14]. The above payoff matrix assumes that the payoff of a block is 1 when both blocks are in residential use and b when a block in commercial use interacts with another in residential use. In other situations, the payoff is 0. In the elementary cellular automata with two neighbors, there are totally eight arrays for the values of a cell and its two neighbors: 111, 110, 101, 100, 011, 010, 001 and 000. When each of the two neighbors interacts with its outward neighbor, there are four possibilities for each array, as shown below. The numbers in the brackets denote the payoffs of the three cells in the simplified prisoner's dilemma game, as shown in the above matrix. The last row shows the sum of the payoffs of the four possibilities for each of the

[11] Nowak, Martin A. and Robert M. May. 1993. "The spatial dilemmas of evolution," *Journal of Bifurcation and Chaos*, 3: 35-78.

[12] Lai, Shih-Kung and Haoying Han. 2012. "On failure of zoning," *International Journal of Society Systems Science*, 4(4): 369-380.

[13] Nowak, Martin A. and Robert M. May. 1993. "The spatial dilemmas of evolution," *Journal of Bifurcation and Chaos*, 3: 35-78.

[14] Wolfram, Stehpen. 2002. *A New Kind of Science*. Champaign, IL: Wolfram Media

three cells arrays. Whether the value of the central cell in each column is zero or one in the next time step is determined by the value of the cell with a larger payoff between the two neighboring cells in the last row.

01110	01100	01010	01000	00110	00100	00010	00000
(b0b)	(bb2)	(2b12b)	(2b23)	(2bb)	(22b2)	(322b)	(333)
01111	01101	01011	01001	00111	00101	00011	00001
(b00)	(bb1)	(2b1b)	(2b22)	(2b0)	(22b1)	(32b)	(332)
11110	11100	11010	11000	10110	10100	10010	10000
(00b)	(0b2)	(b12b)	(b23)	(1bb)	(12b2)	(222b)	(233)
11111	11101	11011	11001	10111	10101	10011	10001
(000)	(0b1)	(b1b)	(b22)	(1b0)	(12b1)	(22b)	(232)
(2b02b)	(2b4b6)	(6b46b)	(6b810)	(64b2b)	(68b6)	(1086b)	(101210)

It can be easily shown that when the value ofb falls in different intervals as shown in Table 2, different transition rules reign of the elementary cellular automata and the space-time plots corresponding to these rules show different patterns. According to Wolfram's classification, a Class I rule results in homogeneous pattern with all white or black cells (see Figures 1, 2, and 5), reminiscent of a single use neighborhood, while a Class Ⅱ rule comes up with a pattern of fixed structures (see Figures 3 and 4), reminiscent of a mixed use neighborhood. Note that in this particular example a white cell symbolizes "1" as commercial use and a black cell represents "0" as residential use and that each transition rule in the elementary CA corresponds to a gua in the I Ching and depends in turn on the value of b, which could be controlled through policy making, such as imposing a price system on land[15].

Table2 Transition rules corresponding to the values of b

Ruleb	Binary Code	Wolfram's Classes
b≤2/3	10000000	Class 1
2/3<b<3/4	10100000	Class1
3/4≤b<3/2	10100100	Class 2
3/2≤b≤5/3	11101100	Class 2
b>5/3	11111110	Class 1

Figure 1 The space-time plot of the rule of 10000000

Figure 2 The space-time plot of the rule of 10100000

〔15〕 Hopkins, Lewis D. 1974. "Plan, projection, policy — mathematical programming and planning theory", *Environment and Planning A*, 6: 419-430.

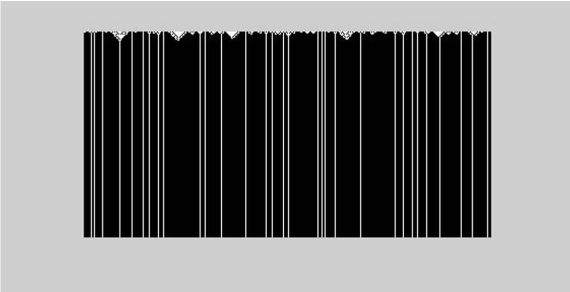


Figure 3 The space-time plot of the rule of 10100100

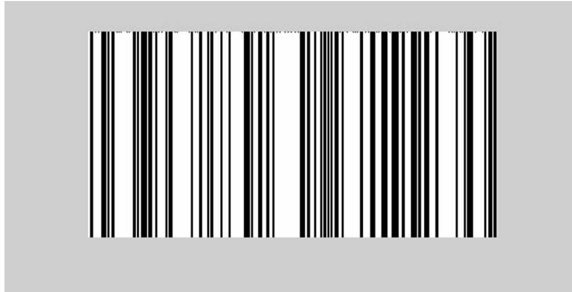


Figure 4 The space-time plot of the rule of 11101100

Figure 5 The space-time plot of the rule of 11111110

4 Discussion

It is well known that the 256 rules of the elementary cellular automata can be classified into four classes[16]. The space-time plots of the transition rules in Class I show homogeneous patterns in which shortly after a few time steps, either white or black cells dominate the space-time configurations, as manifested in Figures 1, 2, and 5. In the case of the space-time plots of the transition rules in Class Ⅱ, fixed structures of vertical stripes emerge as exemplified by Figures 3 and 4. The space-time plots of the transition rules in Class Ⅲ are chaotic structures in that no regularity can be found in the space-time configurations as shown in Figure 6. The space-time plots of the transition rules in the final class, Class IV, are most interesting in that complex structures emerge in a background of seemingly chaos as shown in Figure 7.

What would cities look like if they could be modeled by the elementary cellular automata as depicted in Section 3? Our daily experiences tell us that cities cannot be described by the transition rules in Class I in which nothing is alive. Cities are full of lively agents with changing activities and physical environments. They cannot be depicted by the transition rules in Class Ⅱ in which structures are fixed. Social and physical structures in cities emerge and vanish over time. Cities might look apparently chaotic, but at the same time there is order embedded in them (Lai and Han, 2014); otherwise, we would be at a loss to achieve what we intended to do in cities. Therefore, cities cannot be described by the transition rules in Class Ⅲ. The only transition rules left that may be able to model cities are the ones in Class IV. We argue that cities are inherent orderly in a background of chaos, order coming out of chaos. In Section 3, the transition rules derived from our model fall only in

[16] Wolfram, Stehpen. 2002. *A New Kind of Science*. *Champaign*, IL: Wolfram Media

Classes I and Ⅱ. We suspect that this might be caused by the narrow range of the parameter b in relation to one in the spatial game. With land prices in the real spatial games, we believe the spatial configurations would be more complex in which the space-time plots in Class IV would come to existence. Regardless, the complex structures of Class IV are rare in relation to the universe of all possible transition rules in the elementary cellular automat. The implication is that the probability that cities would emerge might be very small.

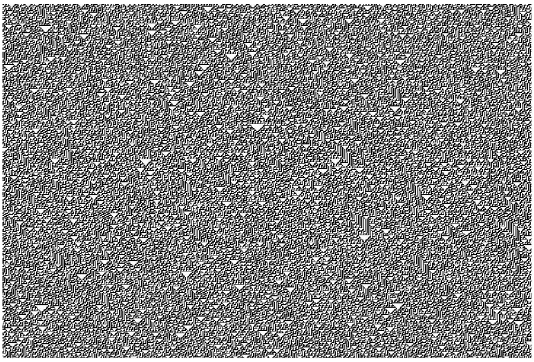


Figure 6 An Example of Chaotic Structure (Rule 30: Adopted from Wolfram[17])

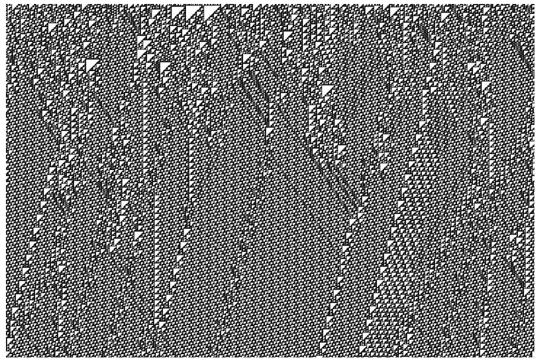


Figure 7 An Example of Complex Structure (Rule 110: Adopted fromWolfram[18])

5 Conclusions

We have shown how the 64guas in the I Ching (Binary System 1) are related to the 256 transition rules in the elementary cellular automata (Binary System 2) to create a consolidated Binary System 3 of Complex I. We also have demonstrated how the elementary cellular automata can be applied to link spatial games to the transition rules of the binary system to model urban change. This research is intended to serve as a starting point where the literatures derived from the two binary systems, the I Ching and the elementary cellular automata, can be linked in the hope of prompting useful insights into modeling the complex world we find ourselves. Much remains to be done.

[17] Wolfram, Stehpen. 2002. *A New Kind of Science*. *Champaign*, IL: Wolfram Media

[18] Wolfram, Stehpen. 2002. *A New Kind of Science*. *Champaign*, IL: Wolfram Media

中文题目:

复杂城市系统的二元建模

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提要:城市是非线性的复杂系统而无法用传统的分析模式来描述。我们需要新的建模技术来了解复杂城市系统运作背后的机理。本文介绍两个二元系统:易经以及基本元胞自动机,并说明它们之间的关系。本文尝试应用此二元模式与空间博弈结合,来解释土地的混合使用如何产生。这个建模技术能帮助我们洞悉城市如何运作。

关键词:易经;元胞自动机;复杂科学;城市